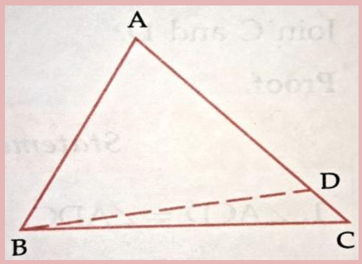
General instructions for Students: Whatever be the notes provided, everything must be copied in the Maths copy and then do the HOMEWORK in the same copy.

INEQUALITIES IN A TRIANGLE

Theorem 10.3

Statement: *If two sides of a triangle are unequal, then the longer side has greater angle opposite to it.*



Given: In ΔABC, $AC > AB$

To Prove: $\angle ABC > \angle ACB$

Construction: Take a point D on AC such that $AD = AB$. We join BD.

Proof: In ΔABD,

$AB = AD$ (By const.)

$\therefore \angle ABD = \angle ADB$ ($\angle$s opp. to equal sides are equal) …………….(i)

Now, $\angle ADB$ is an exterior angle of ΔBCD

$\therefore \angle ADB = \angle ACB + \angle CBD$ (An exterior angle prop. of a triangle)

Or, $\angle ADB > \angle ACB$

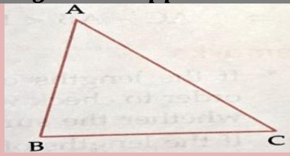
Or, $\angle ABD > \angle ACB$ [By (i)] ……………..(ii)

But, $\angle ABC > \angle ABD$ …………..(iii)

$\therefore$ From (ii) and (iii), $\angle ABC > \angle ACB$ **Proved.**

Theorem 10.4

Statement: *If two angles of a triangle are unequal, then the greater angle has longer side opposite to it.*



Given: In ΔABC, $\angle B > \angle C$

To Prove: $AC > AB$

Proof: We have the following possibilities only. (i) $AC = AB$ (ii) $AC < AB$ (iii) $AC > AB$

Out of these possibilities exactly one must be true.

Case I If possible, let $AC = AB$. then,

$AC = AB \Rightarrow \angle ABC > \angle ACB$ ($\angle$s opp. to equal sides are equal)

Contradicts

Our supposition is wrong

$\therefore AC \neq AB$

Triangles

Case II If possible, let $AC < AB$, then, $\angle B < \angle C$ (longer side has greater angle opp. to it)

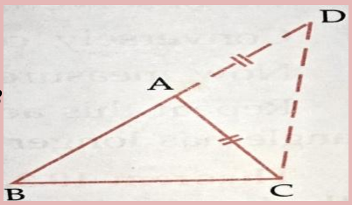
Contradicts

Our supposition is wrong

Case III Thus, we are left with the only possibility, $AC > AB$, which must be true.

Hence, $AC > AB$ **Proved.**

Theorem 10.5 Statement: The sum of any two sides of a triangle is greater than the third side.



Given : In ΔABC.

To Prove : (i) $AB + AC > BC$ (ii) $AB + BC > AC$ (iii) $BC + AC > AB$

Construction : Produced BA to D such that $AD = AC$. We join CD.

Proof : In ΔACD, $AC = AD$ $\therefore \angle ACD = \angle ADC$ ($\angle$s opp. to equal sides are equal)

$\Rightarrow \angle BCA + \angle ACD > \angle ADC$

$\Rightarrow \angle BCD > \angle BDC$ $[\because \angle ADC = \angle BDC]$

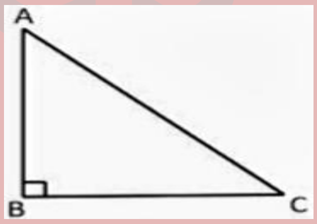
$\Rightarrow BD > BC \Rightarrow BA + AC > BC$ $(\because AD = AC)$

Hence, $AB + AC > BC$

Similarly, $AB + BC > AC$ and $BC + AC > AB$ **Proved.**

EXERCISE – 10.4

Q. No. 2 Show that in a right – angled triangle, the hypotenuse is the longest side.



Solution : In ΔABC, $\angle B = 90°$

We know that, $\angle A + \angle B + \angle C = 180°$ (Angles sum prop. of a triangle)

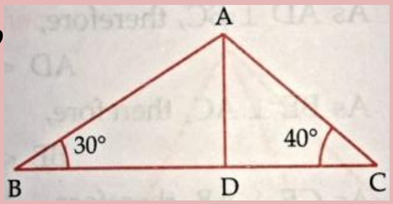
$\therefore \angle A + 90° + \angle C = 180°$ $(\because \angle B = 90°)$

$\Rightarrow \angle A + \angle C = 90°$ $\therefore \angle B > \angle A$ and $\angle B > \angle C$

$\therefore AC > BC$ and $AC > AB$ (Greater angle has longer side opp. to it)

Hence, AC (hypotenuse) is the longest side. **Proved**

Q. No. 6 A figure given alongside, $\angle B = 30°$ and $\angle C = 40°$ and the bisector of $\angle A$ meets BC at D.



Show that : (i) $BD > AD$ (ii) $DC > AD$ (iii) $AC > DC$ (iv) $AB > BD$

Solution : In ΔABC, $\angle A + \angle B + \angle C = 180°$ (Angles sum prop. of a triangle)

$\therefore \angle A + 30° + 40° = 180°$ (Given: $\angle B = 30°$ and $\angle C = 40°$)

$\Rightarrow \angle A = 110°$

$\therefore \angle BAD = \angle CAD = \frac{1}{2} \angle BAC = 55°$

Triangles

$In\ \Delta ABD, \quad \angle BDA = 180° - (30° + 55°) \quad \Rightarrow \quad \angle BDA = 95° \quad (Angles\ sum\ prop.\ of\ a\ triangle)$

$In\ \Delta ACD, \quad \angle CDA = 180° - (40° + 55°) \quad \Rightarrow \quad \angle CDA = 85° \quad (Angles\ sum\ prop.\ of\ a\ triangle)$

$Now. \quad In\ \Delta ABD, \quad \angle BAD > \angle ABD \quad [\because \angle BAD = 55°\ and\ \angle ABD = 30°]$

$\therefore \quad BD > AD \quad (Greater\ angle\ has\ longer\ side\ opp.\ to\ it)$ **Proved (i)**

$In\ \Delta ACD, \quad \angle DAC > \angle ACD \quad [\because \angle DAC = 55°\ and\ \angle ACD = 40°]$

$\therefore \quad DC > AD \quad (Greater\ angle\ has\ longer\ side\ opp.\ to\ it)$ **Proved (ii)**

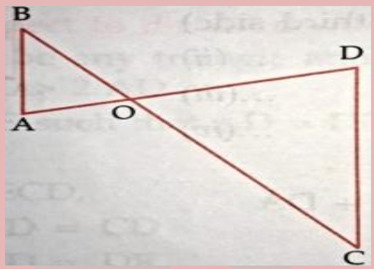
$In\ \Delta ACD, \quad \angle ADC > \angle DAC \quad [\because \angle ADC = 85°\ and\ \angle DAC = 55°]$

$\therefore \quad AC > DC \quad (Greater\ angle\ has\ longer\ side\ opp.\ to\ it)$ **Proved (iii)**

$In\ \Delta ABD, \quad \angle ADB > \angle BAD \quad [\because \angle ADB = 95°\ and\ \angle BAD = 55°]$

$\therefore \quad AB > BD \quad (Greater\ angle\ has\ longer\ side\ opp.\ to\ it)$ **Proved(iv)**

Q.No. 9(a) In the figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



$Solution: In\ \Delta ABO, \quad \angle B < \angle A \quad (Given)$

$\therefore \quad OA < OB \ldots\ldots\ldots\ldots (i) \quad [Greater\ angle\ has\ longer\ side\ opp.\ to\ it]$

$In\ \Delta COD, \quad \angle C < \angle D \quad (Given)$

$\therefore \quad OD < OC \ldots\ldots\ldots\ldots (ii) \quad [Greater\ angle\ has\ longer\ side\ opp.\ to\ it]$

$(i) + (ii) \quad \Rightarrow \quad OA + OD < OB + OC$

$\therefore \quad AD < BC$ **Proved.**

Q.No. 10(i) Is it possible to construct a triangle with lengths of sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

$Solution: By\ the\ Triangle\ Inequality\ Theorem, \quad AB + AC > BC, \quad AB + BC > AC \quad and \quad BC + AC > AB$

$Here, \quad AB = 3\ cm, \quad BC = 4\ cm\ and\ AC = 7\ cm \quad (Say)$

$$3 + 7 > 4, \quad 3 + 4 = 7 \quad and \quad 4 + 7 > 3$$

$No, we\ can't\ construct\ a\ triangle\ with\ lengths\ of\ sides\ as\ 4\ cm,\ 3\ cm\ and\ 7\ cm$

$because \quad 3 + 4 = 7. \quad Here.\ it\ is\ not\ possible\ to\ construct\ a\ triangle.$

$(\because The\ sum\ of\ any\ two\ sides\ of\ a\ triangle\ is\ greater\ than\ the\ third\ side)$

HOMEWORK

EXERCISE – 10.4

QUESTION NUMBERS: 3, 4, 7, 8(a), (c) and 10(iii)

Triangles